OJASS ISSN: 2315-5701

OSCOTECH JOURNAL OF ARTS AND SOCIAL SCIENCES (OJASS)

A BI-ANNUAL ACADEMIC JOURNAL OF THE FACULTY OF MANAGEMENT SCIENCES,

OSUN STATE COLLEGE OF TECHNOLOGY, ESA OKE

MARCH 2017 EDITION

http://ojass.oscotechesaoke.edu.ng/en/

Vol. 4 No. 1

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A VARIANT MODEL FOR HEAT CONDUCTION IN SOLID ROD USING FOURTH ORDER RUNGE-KUTTA METHOD.

O.B. Aluko
Department of Maths and Statistics,
Osun State Polytechnic Iree.
Olabisialuko9.oa@gmail.com

I.T.Mohammed
Department of Maths and Statistics,
Osun State Polytechnic Iree.

O.Asabi
Department of Maths and Statistics,
Osun State Polytechnic Iree.

Abstract

This paper analyzed the equation describing the conduction of heat in a solid rod as proposed by Fourier's law of conduction. The paper aims at proposing a variant model for describing the conduction of heat in a solid rod using Runge-Kutta method. The research was carried out by studying the effect of heating a rod and deriving a variant model using Runge-kutta method. The derived model was then simulated using matlab to evaluate its performance. The results show that when the time of heating is increased from 1.25 secs to 6 secs at an interval of 0.25secs, the temperature of the rod increased from 5000 kelvin to 5011.25 kelvin which affirms that the derived Runge-Kutta based model effectively described the conduction of heat in a solid rod.

Keywords: Fourier law of conduction, heat transfer, thermodynamics and RK4.

Introduction.

Heat transfer is defined as energy-in-transit due to temperature difference. Heat transfer takes place whenever there is a temperature gradient within a system or whenever two systems at different temperatures are brought into thermal contact. Heat, which is energy-in-transit, cannot be measured or observed directly, but the effects produced by it can be observed and measured. Since heat transfer involves transfer and/or conversion of energy, all heat transfer processes must obey the first and second laws of thermodynamics. However unlike thermodynamics, heat transfer deals with systems not in thermal equilibrium and using the heat transfer laws it is possible to find the rate at which energy is transferred due to heat transfer. Conduction heat transfer phenomena are found throughout virtually all of the physical world and the industrial domain. Thermodynamics deals with the end states of the processes and provides no information on the physical mechanisms that caused it. Heat transfer is an example of such a process. A convenient definition of heat transfer is energy in transition due to temperature differences (Chris and Naser 2009). Conduction occurs at molecular level when a temperature gradient exists in a medium, which can be solid or fluid. Heat is transferred along that temperature gradient by conduction. (Aluko, Oduwole and Alaje 2014) presented Numerical Simulation of heat transfer of a lumped mass and analyses heat transfer problem of a lumped mass. A comparative analysis was carried out for investigating the behavior of the result of the existing and proposed models. The comparison shows that the newly proposed model based on Runge-Kutta yielded better results than existing solution.

Understanding the dynamical origin of the mechanisms which underlie the phenomenology of heat conduction has remained one of the major open problems of statistical mechanics ever since Fourier’s seminal work (Fourier 1822). Fourier contribution states the nature and content of the heat conduction process, which is the transient heat diffusion equation, pertaining to the conductive transport and storage of heat in a solid body. The body itself, of finite shape and size, communicates with the external world by exchanging heat across its boundary. Within the solid body, heat manifests itself in the form of temperature, which can be measured accurately. Under these conditions, Fourier's differential equation mathematically describes the rate at which temperature is changing at any location in the interior of the solid as a function of time. Physically the equation describes the conservation of heat energy per unit volume over an infinitesimally small volume of the solid centered at the point of interest. Crucial to such conservation of heat is the recognition that heat continuously moves across the surfaces bounding the infinitesimal element as dictated by the variation of temperature from place to place within the solid and that the change in temperature at a point reflects the change in the quantity of heat stored in the vicinity of the point. It is clear from the above that the notions of temperature, quantity of heat, and transport of heat, as well as the relation between quantity of heat and temperature, are fundamental to Fourier's heat conduction model. It is important to recognize here that these basic notions were still evolving when Fourier developed his equation. Since heat can be readily observed and measured only in terms of temperature, the development of a reliable thermometer capable of giving repeatable measurements was critical to the growth of the science of heat. Objects are made up of matter and matter consists of atoms and particles. Particle that move around in an object possess kinetic energy. Increase in temperature increases the kinetic energy of a particle.

The most basic method of heat transport is conduction. Pure conduction (diffusion) results in a medium with no bulk motion. Here, the microscopic collisions of particles due to a temperature gradient transfer energy from more energetic particles to less energetic particles. (Incorpera and De-Witt 2002) & (Joseph and Preziosi1989) derived the equations describing the conduction of heat in solids have proved to be powerful tools for analyzing not only the transfer of heat, but also an enormous array of diffusion-like problems appearing in physical, chemical, biological, earth and even economic and social sciences. This is because the conceptual mathematical structure of the non-stationary heat conduction equation, also known as the heat diffusion equation, has inspired the mathematical formulation of several other physical processes in terms of diffusion, such as electricity flow, mass diffusion, fluid flow, photons diffusion, etc (Mandelis, 2000; Marín, 2009a). A review on the history of the Fourier´s heat conduction equations and how Fourier´s work influenced and inspired others can be found elsewhere (Narasimhan, 1999). Moreover, the temperature, the basic parameter of Thermodynamics, may not be defined at very short length scales but only over a length larger than the phonons mean free paths, since its concept is related to the average energy of a system of particles (Cahill, et al., 2003;Wautelet & Duvivier, 2007). On the other hand there are some aspects of the heat conduction through solids heated by time varying sources that contradict common intuition of many people, being the subject of possible misinterpretations. The same occurs with the understanding of the role of thermal parameters governing these phenomena.

It is therefore desirable to apply a popular method such as Runge-Kutta method to derive a variant model that can adequately describe the behaviour of heat conduction in a solid rod, which is the aim of carrying out this research.

The Existing Model.

Consider the constant cross-sectional-area rod. Heat diffusion transfers energy along the rod and energy is transferred from the rod to the surrounding by convection. An energy balance on the differential control volume yields

$$\dot{q}\,(x) = \dot{q}\,(x+dx) + \dot{q}_c\,(x) \qquad 1$$

which can be written as

$$\dot{q}\,(x) = \dot{q}(x) + \frac{d}{dx}[\dot{q}(x)]dx + \dot{q}_c(x) \qquad 2$$

which yields

$$\frac{d}{dx}\,[\dot{q}(x)]dx + \dot{q}_c(x) = 0 \qquad 3$$

Heat diffusion is governed by Fourier's law of conduction, which state that

$$\dot{q}(x) = -KA\frac{dT}{dx} \qquad 4$$

Where $\dot{q}(x)$ is the energy transfer rate (J/s)

K is the thermal conductivity of the solid (J/s-m-k)

A is the cross sectional area of the rod (m^2)

$\frac{dT}{dx}$ is the temperature gradient (k/m)

Heat transfer by convection is governed by Newton's law of cooling

$$\dot{q}_c(x) = hA(T - T_a)$$

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Where h is an empirical heat transfer coefficient (J/s-m^2-K)

A is the surface area of the rod (A=Pdx, m^2)

P is the perimeter of the rod (m)

T_a is the ambient temperature (K)

'-'sign is a consequence of 2^{nd} law of thermodynamics (i.e., dT/dx must be negative).

Substituting eqn(4) and (5) into eqn(3)

$$\frac{d}{dx}\left(-KA\frac{dT}{dx}\right)dx + h(Pdx)(T - T_a) = 0 \qquad 6$$

For constant K, A and P eqn(6) yields

$$\frac{d^2T}{dx^2} = \frac{hP}{KA}(T - T_a) = 0$$

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which can be rewritten as

$$T'' - \alpha^2 T = -\alpha^2 T_a \qquad 8$$

Where $\alpha^2 = \frac{hP}{KA}$

Equation (8) is a linear second order boundary-value ordinary differential equation.

General solution of (8) is

$$T(x) = Ae^{\alpha x} + Be^{-\alpha x} + T_a \qquad 9$$

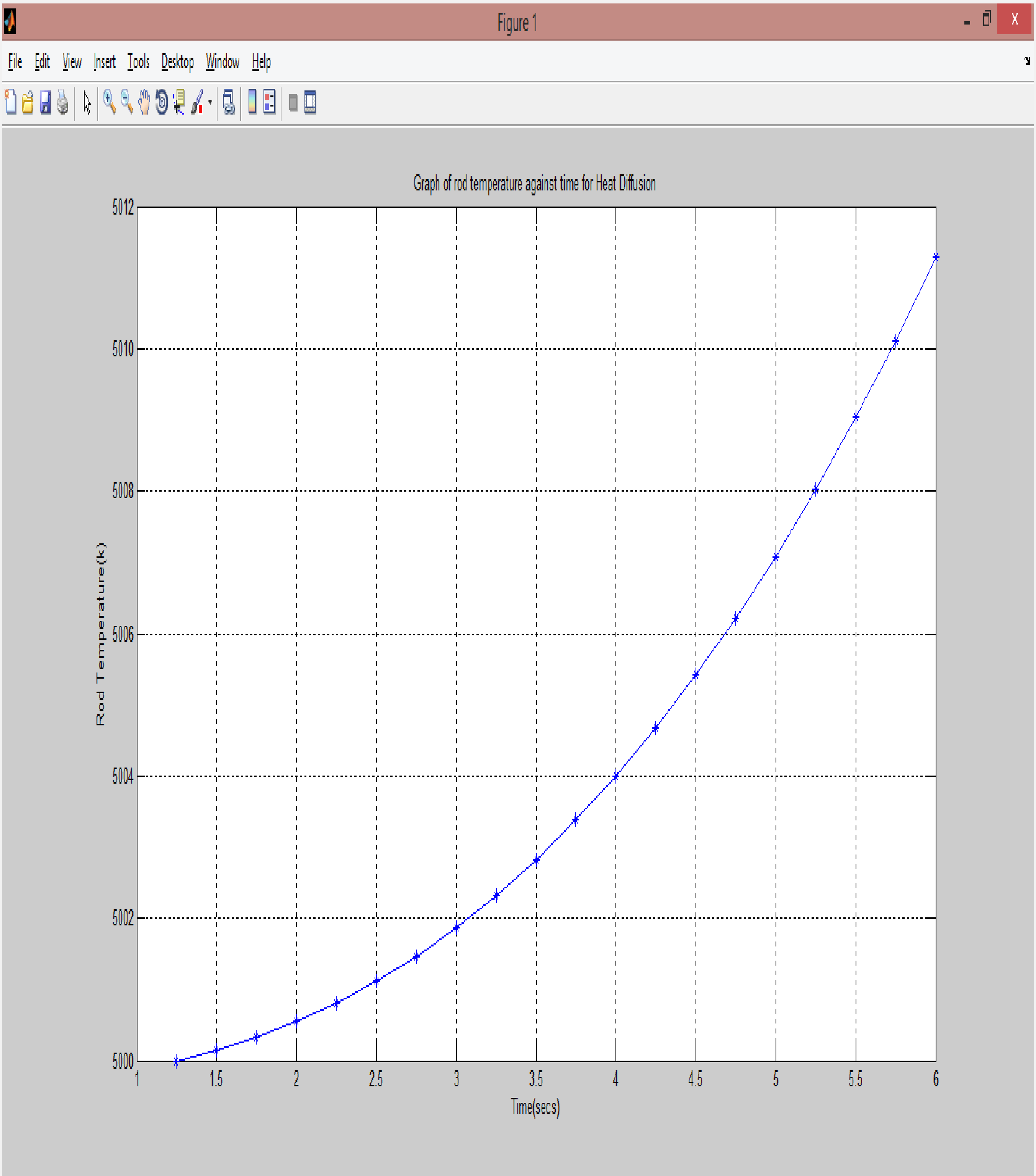
Where

$$A= \frac{(T_2-T_a)-(T_1-T_2)e^{-\alpha L}}{e^{\alpha L}-e^{-\alpha L}} \text{ and } B= \frac{(T_1-T_2)e^{\alpha L}-(T_2-T_a)}{e^{\alpha L}-e^{-\alpha L}} \qquad 10$$

The Derived Variant Model.

The variant model was derived by solving equation (8) above using Runge-Kutta method. The simulation of the solution using matlab is as shown in figure 1 below

Figure 1: Graph of rod temperature against time for heat diffusion.



Result and Discussion.

Figure 1 shows the result of simulating the solution equation (8) by Runge-Kutta using matlab.

From the result, it is observed that as the time of heating the solid rod increases from 1.25 seconds to 6 seconds, the temperature increases from 5000 Kelvin to 5011.25 Kelvin. This affirms that the derived model adequately described the conduction of heat by the rod based on the observed increase in temperature with corresponding increase in time of heating.

Conclusion

It is concluded from the result that the variant model proposed in this work can be used to adequately describe the conduction of heat by a solid rod.

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